UNIVERSITY OF
ILLINOIS LIBRARY
AT URBANA-CHAMPAIGN
BOOKSTACKS

Digitized by the Internet Archive
in 2011 with funding from
University of Illinois Urbana-Champaign

http://www.archive.org/details/methodsofinvesti340kauj

Faculty Working Papers

METHODS OF INVESTIGATING ALTERNATIVE MODELS OF URBAN STRUCTURE

James B. Kau and Cheng F. Lee

#340

College of Commerce and Business Administration
University of Illinois at Urbana-Champaign

FACULTY WORKING PAPERS

College of Commerce and Business Administration

University of Illinois at Urbana-Champaign

October 5, 1976

METHODS OF INVESTIGATING ALTERNATIVE MODELS OF URBAN STRUCTURE

James B. Kau and Cheng F. Lee

#340

METHODS OF INVESTIGATING ALTERNATIVE MODELS OF URBAN STRUCTURE

I. Introduction

This paper provides techniques and insights for choosing among alternative functional forms to explain urban structure. The choice criterion for determining the best predictive model and the impact of specification bias on the predictive power of the model are investigated using data for 50 United States cities.[1]

The negative exponential density function has been widely used to explain the relationship between distance to the central business district (CBD) and population density. The accepted use of this function is explained by its theoretical foundation and the ease of its empirical estimation. Muth [17] using constrained utility maximization derived the necessary and sufficient conditions for the existence of a negative exponential function. The data requirements for estimation are a sample of density values measured at different distances from the urban center. This data being readily available makes the exponential function an attractive relationship to estimate.

Unfortunately, the use of distance as a surrogate for transportation cost and other characteristics of a city may lead to estimated bias in the density gradient. Also the restrictive assumptions required for derivation of the negative exponential function such as a unitary elasticity of substitution and a price elasticity of demand equal to minus one may lead to specification bias. Recently, empirical evidence has indicated that the simplified two variable density model does not provide an adequate description of the density-distance relationship. Kau and Lee [7,8] have demonstrated the existence of specification bias and have used a generalized functional form for the density gradient to reduce the extent of bias. Muth [18] using a constant elasticity of substitution (CES)

production function, numerical analysis and alternative values of the elasticity of substitution between land and structures demonstrated the inappropriateness of an exponential function derived from the Cobb-Douglas production function in predicting the actual distribution of population densities. McDonald and Bowman [14] have studied alternative functional forms and found that the explanatory power of the negative exponential function can be improved in some cases by adding a quadratic distance term, but that total population living in a predefined urban area can be predicted more accurately if the quadratic term is omitted.

The proven probability of possible bias when using an exponential function in estimating the density gradient suggests that alternative functional forms may be appropriate. Density equations derived from CES production functions, while theoretically sound, are difficult to estimate because of limited data (See Fallis [5], Koenker [11], and Muth [18]). The Box and Cox transformation technique [2] used by Kau and Lee [7,8] indicates the possibility of extending the usefulness of the negative exponential function by the addition of a quadratic term. Muth [17] found that twelve of the 46 cities had significant quadratic terms at the 0.10 level and that the deviations from linearity were too frequent to attribute to sampling variability. The quadratic terms were not predominantly positive or negative and they were not associated with city size or region of the country. Thus, Muth found significant variation from linearity, but was unable to draw meaningful conclusions about the role of the quadratic distance term in explaining urban structure. Latham and Yeates [12] developed the use of a negative quadratic exponential and Mills [15] has compared linear and log forms of a distance-density relationship. Kemper and Schmenner [9]

concluded that the exponential functional form was not completely satisfactory in describing the variation of manufacturing densities with distance. Taylor [20] has estimated distance-decline functions of an interaction surface utilizing various transformations of the independent variable distance. The results which apply only to the independent variable indicate that neither the double nor single log transformation is optimal.

In Section II, a Box and Cox [2] transformation technique is used to introduce a functional form parameter to generalize the relationship between the population density and distance for nine alternative specifications. Section III provides an approach to measure the relative skewness and relative kurtosis measurements of density and distance to indicate whether quadratic and/or cubic distance terms with semi or double log specification should be used to determine the relationship between population density and distance. The empirical results and implications of skewness and kurtosis are examined in Section IV. There was significant skewness in the distance variable for all 50 cities and kurtosis was evident in most cities. Section V represents the optimal generalized functional form for each of the nine equations. More than half of the cities exhibited optimal generalized forms not currently used in urban studies. This functional form technique is a more robust approach to account for bias associated with skewness and kurtosis effects.[2] Finally, after the optimal generalized form is determined for each of the nine equations then a generalized format for using the integral of the density function is presented for predicting total population.

II. A Generalized Model

The fact that the actual structure of an urban area differs from the predictions casts considerable doubt on the validity of the exponential model. In general, explanations for the model's poor prediction behavior have come from the assumptions upon which the model is based or from examining the empirical problems in testing the model. Thus, recent studies have concentrated on more elaborate functional forms or extensions of the data. (See Kau and Lee [6] , McDonald and Bowman [14], and Muth [18].)

This study explores specification bias resulting from skewness and kurtosis in the data and from omitted variables. These particular problems are common with the current urban models. Box and Cox [2] have provided a way to reduce specification bias by generalizing the functional form for describing the relationship between the population density and distance. For example, one possible form would be

$$(D_i^{\lambda} - 1)/\lambda = \gamma_0 - \gamma_1 U_i + \varepsilon_i \qquad (1)$$

where γ_0 and γ_1 are regression parameters, λ is a functional form parameter and the disturbance term is normally distributed with zero mean and variance of σ^2. Equation (1) will reduce to the linear form when $\lambda=1$ and the common exponential form when $\lambda \to 0$.

Under the assumption of normality, the probability density function for ε_i in equation (1) is written as

$$f(\varepsilon_i) = (2\pi\sigma^2)^{-1/2} \exp(-1/2 \frac{\varepsilon^2_i}{\sigma^2}). \qquad (2)$$

If the ε's are identically and independently distributed, then the log likelihood function for equation (2) is

$$\text{LogL} = -n/2 \log 2\pi\sigma^2 + (\lambda-1) \sum_{i=1}^{n} \log D_i - \frac{1}{2\sigma^2} \sum_{i=1}^{n} \left\{ \frac{(D_i^\lambda - 1)}{\lambda} - \gamma_o + \gamma_1 U_i \right\}^2. \quad (3)$$

The logarithmic likelihood is maximized with respect to σ^2, γ_o and γ_1 given λ. The maximum likelihood estimates of σ^2 for the given λ, $\hat{\sigma}^2(\lambda)$ is then the estimated variance of the disturbances of regressing $(D_i^\lambda - 1)/\lambda$ on U_i. Replacing σ^2 by $\hat{\sigma}^2(\lambda)$, the maximum log likelihood [Lmax(λ)] for equation (1) is

$$\text{Lmax}(\lambda) = -n/2 - n/2 \log 2\pi\hat{\sigma}^2(\lambda) + (\lambda-1) \sum_{i=1}^{n} \log D_i. \quad (4)$$

The approximate 95% confidence region for λ is $\text{Lmax}(\hat{\lambda}) - \text{Lmax}(\lambda) < 1/2\chi^2(.05) = 1.92.$ (5)

In a study by McDonald and Bowman [14] a number of alternative distance-density relationships were compared. The basis for comparison was the mean standard error of the regression for each form. Secondly they investigated the prediction accuracy for each model. Their empirical results were somewhat inconclusive regarding the best model relative to either explanatory or prediction power.

Most of the alternative functions in the McDonald-Bowman paper will be used in this study as a basis for investigating the proper methodological approach for comparing alternative forms and for determining the best predictive model. The generalized alternative functions used for comparison are

$$\frac{D^\lambda - 1}{\lambda} = \gamma_1 + \gamma_2 u \quad (6)$$

$$\frac{D^\lambda - 1}{\lambda} = \gamma_1 + \frac{\gamma_2(\bar{u} - u)^\lambda - 1}{\lambda} \quad (7)$$

$$\frac{D^{\lambda}-1}{\lambda} = \gamma_1 + \gamma_2 u^2 \tag{8}$$

$$\frac{D^{\lambda}-1}{\lambda}(u) = \gamma_1 + \gamma_2 \log u + \gamma_3 u \tag{9}$$

$$\frac{D^{\lambda}-1}{\lambda} = \gamma_1 + \gamma_2 u + \gamma_3 1/u \tag{10}$$

$$\frac{D^{\lambda}-1}{\lambda} = \gamma_1 + \gamma_2 \frac{(u)^{\lambda}-1}{\lambda} + \gamma_3 u \tag{11}$$

$$\frac{D^{\lambda}-1}{\lambda} = \gamma_1 + \gamma_2 \frac{(u)^{\lambda}-1}{\lambda} + \gamma_3 \frac{(1/u)^{\lambda}-1}{\lambda} \tag{12}$$

$$\frac{D^{\lambda}-1}{\lambda} = \gamma_1 + \frac{u^{\lambda}-1}{\lambda} \tag{13}$$

$$\frac{D^{\lambda}-1}{\lambda} = \gamma_1 + \gamma_2 u + \gamma_3 u^2 \tag{14}$$

The λ values are used to generalize the nine equations according to the Box and Cox technique as indicated by equations (1)-(4). One of the justifications for using this technique is the possibility of skewness in data which is explored in the next section.

III. Specification Bias and Skewness: The Model

There is the possibility of bias when using a functional form which excludes relevant socioeconomic and geographical variables. Many studies suggest that the use of distance from the central business district as a proxy for transportation cost is not suitable because of the existence of multi-center cities. Other studies suggest that the emphasis on distance (i.e., accessibility) leads to inaccurate conclusions about its importance as a determinant of urban structure. Also cities are often not structured in perfect, concentric circles but instead have irregular transportation patterns due to past development, political boundaries and geographical phenomena such as mountains and water.

The possibility of bias when excluding socioeconomic and geographical variables can be determined by the degree of symmetry and normality in the data for population density and distance. Following Snedecar and Cochran [19] the relative skewness and kurtosis can be used to test the degree of symmetry of a distribution and are defined as

$$g_1 = k_3/k_2^2 \tag{15}$$

$$g_2 = k_4/k_2^2 \tag{16}$$

where g_1 = the relative skewness;

g_2 = the relative kurtosis;

$$K_2 = E[(x_i - \bar{x})^2]; \tag{17}$$

$$K_3 = E[(x_i - \bar{x})^3]; \tag{18}$$

$$K_4 = E[(x_i - \bar{x})^4]; \text{ and} \tag{19}$$

x_i = either distance u_i or density D_i.

The standard errors used to test the significance of g_1 and g_2 can be defined as

$$Sg_1 = [6n(n-1)/(n-2)(n+1)(n+3)]^{1/2} \tag{20}$$

$$Sg_2 = [24n(n-1)^2/(n-3)(n-2)(n+3)(n+5)]^{1/2} \tag{21}$$

where

Sg_1 = the standard error for g_1;

Sg_2 = the standard error for g_2; and

n = the sample size.

Equations (15), (16), (20), and (21) will be used to test the degree of symmetry and the degree of normality for population density and distance with and without the log transformation.

The impact of skewness is demonstrated by using alternative functional forms based on the generalized relationship between population density and distance developed by Kau and Lee [8]. This generalized relationship can be defined as

$$\frac{D^{\lambda}(u) - 1}{\lambda} = \frac{[C^{\lambda} - 1]}{\lambda} + \gamma \bar{u} - \gamma u \qquad (22)$$

where λ = the functional form parameter;

C = a parameter which is a function of rent on nonurban land, price elasticity and income, rate of return on capital, and relative factor shares;

$\bar{u}$ = the distance from the center to the edge of the urban area; and

u = distance from the urban center.

By the Taylor expansion, the left hand side of equation (22) can be approximated as

$$\frac{D^{\lambda}(u) - 1}{\lambda} = \log D(u) + 1/2!\lambda \{\log D(u)]^2 + 1/3!\lambda^2 [\log D(u)]^3 + \ldots \qquad (23)$$

In addition, from equation (22) it can be shown that

$$\log D(u) = 1/\lambda \log[(C^{\lambda} + \gamma\bar{u}) - \lambda\gamma u]$$

$$= 1/\lambda \log[C^{\lambda} + \gamma\bar{u}] - \frac{\gamma u}{C_o^{\lambda}+\gamma\bar{u}} + \frac{(\lambda\gamma)^2}{[(C_o+\gamma\bar{u})]^2} \frac{u^2}{2!} + \ldots \qquad (24)$$

Substituting (23) and (24) into (22) and rearranging terms, equation (22) approximately reduces to

$$\log D(u) = \beta_o - \beta_1 u + \beta_2 u^2 + \beta_3 u^3 + \ldots + \qquad (25)$$

Equation (25) reduces to the negative exponential function when $\lambda \to 0$, which occurs when the price elasticity of housing demand approaches minus one. Equation (25) derived from the generalized density gradient reveals the technique used to capture the effect of omitted variables. Following Theil [21] if the relative skewness is significant then the coefficients associated with distance (U_i) will be biased unless the coefficients associated with U_i^2 are not significant.

Similarly the coefficient associated with U_i will again be biased when the relative kurtosis and the coefficient associated with U_i^3 are significant. This study examines the impact of skewness and kurtosis by using alternative functional forms based on the addition of quadratic and cubic terms to the standard negative exponential form. The relationships between population density and distance used are

$$\log D_i = a_o - a_1 u_i + \varepsilon_{1i} \tag{26}$$

$$\log D_i = b_o - b_1 u_i + b_2 u_i^2 + \varepsilon_{2i} \tag{27}$$

$$\log D_i = c_o - c_1 u_i + c_2 u_i^2 + c_3 u_i^3 + \varepsilon_{3i} \tag{28}$$

$$\log D_i = d_o - d_1 \log U_i + \varepsilon_1 \tag{29}$$

$$\log D_i = e_o - e_1 \log U_i + e_2 (\log U_i)^2 + \tag{30}$$

$$\log D_i = f_o - f_1 \log U_i + f_2 (\log U_i)^2 + f_3 (\log U_i)^3 \tag{31}$$

where

D_i = population density;

U = the distance from the central business district;

a,b,c,d,f,I = regression parameters; and

ε = disturbance terms.

These alternative forms are used with data from 50 cities to test the theoretical hypothesis developed in this section.

IV. Specification Bias and Skewness: Empirical Results

The relative skewness and kurtosis for distance, density, log distance, and log density were calculated using equations (15) and (16). The results listed in Table 1 in the Appendix (available from the authors) indicate that distance and density data for all 50 cities are significantly different from a normal distribution. It was also found that log transformations are not always useful in removing the skewness

and kurtosis effects. Logging distance and density is an example in some cases of over-transformation.[3]

The impact of skewness and kurtosis was investigated using the functional relationships between population density and distance described in equations (26)-(31). The analysis demonstrates that distance squared (U^2) and distance cubed (U^3), or a combination thereof, is important in explaining the variation of density in some cities when using equations (26), (27) and (28), semi-log. If the linear specifications are used, the importance of the higher order terms of distance decreases sharply. The summary results for specifications (26)-(31) are listed in Table 2 in the Appendix (available from the authors). These results imply that the double-log specification without higher order distance term is superior to the semi-log specification without higher order distance term if the distribution of distance is not normally distributed. The quadratic terms are significant in approximately 20 cities and the cubic in 14 cities depending on the particular specification. Incidentally, the linear specification is not acceptable unless both population and distance are normally distributed.

The semi-log specification is the most widely used functional form. Thus the implications of higher order distance terms are explored. Taking the derivative of log D(u) with respect to distance for equation (26), (27), and (28), respectively, we obtain

$$\frac{\partial \log D(u)}{\partial u} = -a_1 \tag{32}$$

$$\frac{\partial \log D(u)}{\partial u} = -b_1 + b_2 U_i \tag{33}$$

$$\frac{\partial \log D(u)}{\partial u} = -c_1 + c_2 U_i + c_3 U_i^2. \tag{34}$$

These relationships imply that the absolute value of distance is of some importance in explaining the variation of density.

These empirical results have shed more light on the functional relationship between population density and distance. It is shown that the higher order distance terms have some importance in explaining the variation of population density. The phenomenon is essentially due to either the population data or the distance data not being normally distributed.

If the relative skewness and kurtosis for distance is significant and the regression for either U^2 or U^3 is also significant, then omitting U^2 or U^3 will bias the estimated density gradient. The direction of the bias is dependent upon the signs of auxiliary regression coefficient and the regression coefficient association associated with either U^2 or U^3 term.[4] It is evident that skewness and kurtosis is significant in many cities. In the next section a Box and Cox functional form technique is used to eliminate the bias results from skewness.

V. Empirical Estimates of the Optimal λ for the Nine Alternative Specifications

The data for this study consist of a random sample of 45 census tract densities measured u miles from the central business district for each of 50 United States cities in 1970.

The appropriate power transformation for equations (6)-(14) is determined by using equations (1)-(4) and iterating λ between -.50 and 1.50 at intervals of 0.1. The maximum likelihood estimates of these iterations for each of 50 cities indicate that the simple functional forms assumed by McDonald and Bowman are inappropriate.

Table 1 on the following page indicates that approximately 50% of the cities have λ significantly different from zero at the .05 level. Using the critical value of 1.92 from equation (5) the results indicate that the logarithmic specifications dominate the linear specifications, but it is difficult to differentiate the semi-log from the double-log functional form since their log

TABLE 1

Estimated λ Values for Nine Alternative Specifications

Equation	Number of Cities $\lambda \neq 0$	Number of Cities $\lambda \neq 1$
6	20	42
7	27	31
8	31	39
9	28	39
10	28	40
11	37	39
12	33	42
13	24	41
14	26	42

likelihood values are similar. The results of this approach demonstrate the inappropriateness of the nine alternative functional forms as given. The results would suggest that the functional form varies among cities. (The optimal λ value and corresponding confidence intervals for all nine specifications are available from the authors.) For the nine alternative specifications there is no statistical justification for preferring a priori one specification over another. The variation among all functional forms for all cities is of enough significance to warrant empirical investigation for each particular case. In the next section techniques are provided for choosing among alternative specifications.

VI. Prediction and Specification

McDonald and Bowman [14] using a suggestion by Clark [3] test the reliability of a population density function in predicting total population by computing the integral of the function within the appropriate limits. For example, the negative exponential is integrated from $\bar{u}$, the radius of the

central business district to $\underline{u}$, the radius of the urbanized area. Assuming $\underline{u}$ equal to zero, their resulting integral for the negative exponential is

$$N = \phi \exp^{\beta_1} (\beta_2)^{-2} [1 - (1-\beta_2\bar{u})\exp(\beta_2\bar{u})]. \tag{35}$$

The results for this study indicate that the negative exponential form is inappropriate in 40% of the cities. This would mean that the corresponding integral [equation (35)] would be an incorrect form for these cities. This study uses the generalized density gradient which is the exponential function when $\lambda \to 0$, to derive a generalized integral for prediction.

$$D(u) = [\beta_1 + \beta_2 u]^{1/\lambda} . \tag{36}$$

Equation (36) when integrated, is

$$N = \frac{\phi(\beta_2)^{-2}}{(1/\lambda+1)(1/\lambda+2)} \left\{ [\beta_1 + \beta_2\bar{u}]^{1/\lambda+1} (\beta_2\bar{u}\ (1/\lambda+1)-\beta_1) + \beta_1^{1/\lambda+2} \right\}. \tag{37}$$

Thus if $\lambda \not\to 0$, the resulting bias in using equation (35) relative to (37) is evident. All of the McDonald-Bowman functional forms have been generalized with the corresponding integral and can be found in the appendix.

In summary, this paper provides a generalized procedure to test alternative functional forms. The empirical results from using this technique indicate that all of the nine alternative density-distance forms used exhibited specification bias for at least 50% of the cities studied. It was shown that the bias was most likely the result of skewness or kurtosis in the data or to omitted variables. A Box and Cox transformation technique was used to reduce specification and to give insight into the correct functional forms. For most cities, the optimal λ value indicated that either the linear ($\lambda = 1$) or the log ($\lambda \to 0$) specification was inappropriate. Finally, generalized integration of the optimal density-distance formulation was provided to incorporate the optimal λ value into the integral population prediction technique suggested by Clark [3] and used by McDonald and Bowman.

Footnotes

1. Population data for the tracts are from the U.S. Bureau of Census, 1970. Census of Population and Housing Census Tracts, PHC (1). Areas in square miles were measured with a polar planimeter using tract maps. Distance in miles was measured with a ruler on the tract maps from the center of the CBD to the center of the tract. Density is in terms of population per square mile.

2. Box and Cox [2] have shown that the transformation technique can be used to reduce or remove the bias resulting from: 1) skewness effect, 2) heteroscedasticity , and 3) interaction effect.

3. The impacts of skewness effect on the capital market research have received a lot of attention. See, for example, Lee [13] and Fagler and Radcliff [4].

4. See Theil [21], p. 550, for details. This provides insight into why McDonald and Bowman [14] concluded that a quadratic term reduces the mean standard error of estimation.

References

1. Allen, David M., "Mean Square Error of Prediction as a Criterion for Selecting Variables," Technometrics, Vol. 13, No. 3, August 1971.

2. Box, G.E.P. and D. R. Cox, "An Analysis of Transformation," Journal of the Royal Statistical Society, Series B, 26, 1964, pp. 211-243.

3. Clark, Colin, "Urban Population Densities," Journal of the Royal Statistical Society, Series A, 114, 1951, pp. 490-496.

4. Fagler, H. R. and R. C. Radcliff, "A Note on Measurement of Skewness," Journal of Financial and Quantitative Analysis, Vol. 9, 1974, pp. 211-243.

5. Fallis, George. The Technology of Production and the Location of Employment in Urban Areas, Ph.D. Dissertation, Princeton University, March 1975.

6. Kau, James B. and C. F. Lee, "Capital-Land Substitution and Urban Land Use," Journal of Regional Science, 1, April 1976.

7. _____, "The Functional Form in Estimating the Density Gradient: An Empirical Investigation," Journal of the American Statistical Association, June 1976.

8. _____, "A Generalized Functional Form Approach to Investigate the Density Gradient and the Price Elasticity of Demand for Housing," Urban Studies, Vol. 13, No. 1, February 1976.

9. Kemper, P. and R. Schmenner, "The Density Gradient for Manufacturing Industry," Journal of Urban Economics, Vol. 4, No. 1, 1974, pp. 410-427.

10. Klein, L. R. A Textbook of Econometrics, 2nd ed., Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1974.

11. Koenker, R., "An Empirical Note on the Elasticity of Substitution Between Land and Capital in a Monocentric Housing Market," Journal of Regional Science, Vol. 12, No. 2, 1972, pp. 299-305.

12. Latham, R. F. and M. H. Yeates, "Population Density Growth in Metropolitan Toronto," Geographical Analysis, Vol. 2, April 1970, pp. 177-185.

13. Lee, Cheng F., "Functional Form Skewness Effect and the Risk-Return Relationship," Journal of Financial and Quantitative Analysis, March 1977, forthcoming.

14. McDonald, J. and H. W. Bowman, "Some Tests of Alternative Urban Population Density Functions," Journal of Urban Economics, Vol. 3, No. 3, July 1976.

15. Mills, E. S., "Urban Density Functions," Urban Studies, Vol. 7, No. 1, 1970, pp. 5-20.

16. _____. Urban Economics, Glenview, Illinois: Scott, Foresman and Co., 1972.

17. Muth, R. Cities and Housing, Chicago: University of Chicago Press, 1969.

18. _____, "Numerical Solution of Urban Residential Land-Use Models," Journal of Urban Economics, Vol. 2, No. 4, October 1975.

19. Snedecar, G. W. and W. G. Cochran. Statistical Methods, 5th ed., Iowa State College Press, Ames, 1957.

20. Taylor, P. J., "Distance Transformation and Distance Decay Functions," Geographical Analysis, Vol. 3, July 1971, pp. 221-238.

21. Theil, H. Principles of Econometrics, New York: John Wiley & Sons, Inc., 1971.

APPENDIX

Table 1

Relative Skewness and Kurtosis for Distance, Density, Log Distance and Log Density for Fifty Cities in 1970

City	Skewness	Kurtosis	City	Skewness	Kurtosis
Akron			Baltimore		
Distance	.60248*	- .54773	Distance	1.2275*	1.8676*
Density	5.77005*	34.0661*	Density	2.26813*	5.4291*
Log Distance	- .31390*	- .75111*	Log Distance	- .92594*	- .19897*
Log Density	.40348*	2.94229*	Log Density	- .63919*	- .71319
Birmingham			Boston		
Distance	.69228*	- .35314*	Distance	6.8816*	- .08242
Density	1.20879*	2.13379*	Density	1.51088*	2.5534*
Log Distance	- .44701*	- .54958	Log Distance	- .71035*	.38354
Log Density	- .90325*	.00117	Log Density	- .17098	- .06769
Buffalo			Chicago		
Distance	.44711*	- .62506	Distance	.38057*	- .60507*
Density	.77899*	- .05077	Density	2.2146*	6.87321*
Log Distance	- .60645*	- .42735	Log Distance	- .55402*	- .02810
Log Density	- .71978*	.009667	Log Density	.24677	.062536
Cincinnati			Cleveland		
Distance	.19006*	- .01033*	Distance	.60627*	.196779
Density	2.34005*	7.83408*	Density	1.35035*	2.68928*
Log Distance	- .80753*	- .18977	Log Distance	- .54285*	- .34039*
Log Density	- .35485	.64203	Log Density	- .58575*	.74304*
Columbus			Dallas		
Distance	.95694*	1.13366*	Distance	- .12327*	- 1.18543*
Density	.50876*	- .40205	Density	.48808*	- .6632
Log Distance	- .14552*	- .43500	Log Distance	- .88175*	- .38359
Log Density	- .78046*	.12151	Log Density	- 1.11629*	.53351

Table 1 (Continued)

City	Skewness	Kurtosis	City	Skewness	Kurtosis
Dayton			Denver		
Distance	1.08178*	.99022*	Distance	.25244	- .78221
Density	.94950*	.39393	Density	2.40837*	6.6287*
Log Distance	- .06736	- .83621*	Log Distance	- .82158*	.10830
Log Density	- .40200	- .40598	Log Density	- .645579*	1.97102*
Detroit			Flint		
Distance	1.24740*	2.3822*	Distance	.83175*	.26620
Density	1.117235*	1.7775*	Density	1.54232*	2.39198*
Log Distance	- .55622*	.05111	Log Distance	- .29929	- .48569
Log Density	-2.21335*	7.9329*	Log Density	- .59155*	.25275
Fort Worth			Houston		
Distance	.64507*	- .7556*	Distance	1.44327*	3.4373*
Density	.29846	- .697237	Density	.37681*	- .29552
Log Distance	- .414702*	- .59888	Log Distance	- .51309*	- .018176
Log Density	-1.2541*	1.4707*	Log Density	- .13988	1.25538*
Jacksonville			Kansas City		
Distance	.800874*	.36289	Distance	.62723*	- .1422
Density	1.55177*	2.33606*	Density	.66008*	- .20538
Log Distance	- .83164*	.45564	Log Distance	- .68604*	- .17800
Log Density	- .69533*	- .005756	Log Density	- .6524*	- .66316
Los Angeles			Louisville		
Distance	.60549*	- .20864	Distance	1.31313*	1.4959*
Density	1.66259*	2.73937*	Density	1.00654*	.65775
Log Distance	- .90148*	.99063*	Log Distance	- .01711	- .26088
Log Density	.28458	.10700	Log Density	- .5462*	- .06393
Memphis			Miami		
Distance	.29995	-1.07992*	Distance	.10907	-1.16357*
Density	1.09414*	1.4438*	Density	1.85869*	3.64200*
Log Distance	- .40518*	- .99368*	Log Distance	- .92235*	.24440
Log Density	-1.0268*	1.5507*	Log Density	- .24394	.14974

Table 1 (Continued)

City	Skewness	Kurtosis	City	Skewness	Kurtosis
Milwaukee			Nashville		
Distance	.484318*	- .55450	Distance	.72997*	- .39005
Density	1.09389*	.401838	Density	1.67061*	3.59699*
Log Distance	- .19276	- .84503*	Log Distance	- .10191	- .11472
Log Density	- .21518	- .28514	Log Density	- .49207*	- .34004
New Haven			New Orleans		
Distance	.77360*	.34085	Distance	.814249*	- .032801
Density	1.18002*	.76518*	Density	2.30831*	6.77695*
Log Distance	- .60665*	- .16238	Log Distance	- .22316	- .83204*
Log Density	- .54092*	- .47293	Log Density	.230587	- .41636
Oklahoma City			Omaha		
Distance	.64622*	.04577	Distance	.94145*	.54554
Density	.09555	- .39707	Density	4.72582*	25.4925*
Log Distance	-1.06889*	1.7980*	Log Distance	- .42542*	- .19051
Log Density	1.3761*	1.04288*	Log Density	- .48509*	2.9560*
Philadelphia			Phoenix		
Distance	.72685*	.06946	Distance	.31172	- .53400
Density	.63941*	- .80382*	Density	- .30918	- .53084
Log Distance	- .5518*	- .12266	Log Distance	- .11043	1.2777*
Log Density	- .5565*	- .41721	Log Density	-2.04418*	4.4566*
Pittsburgh			Portland		
Distance	1.0262*	.7413*	Distance	.62103*	- .01342
Density	.49395*	- .77308*	Density	.68824*	.18345
Log Distance	- .09476	- .51867	Log Distance	- .77045*	.27682
Log Density	- .40404*	- .479649	Log Density	- .55896*	- .46772
Providence			Richmond		
Distance	1.0269*	.04832	Distance	.75468*	.59634
Density	.61356*	- .55856	Density	2.3797*	6.05594*
Log Distance	- .21298	- .36879	Log Distance	- .44551*	- .69832
Log Density	- .93629*	.51367	Log Density	- .95728*	2.2185*

Table 1 (Continued)

City	Skewness	Kurtosis	City	Skewness	Kurtosis
Rochester			Sacramento		
Distance	.77505*	- .65650	Distance	.98407*	.21680
Density	.58491*	- .983433*	Density	.27622	- .67225
Log Distance	.03581	-1.2794*	Log Distance	- .57395*	- .00421
Log Density	- .84154*	- .24021	Log Density	-1.36055*	1.62671*
Salt Lake City			San Antonio		
Distance	.7789*	- .51426	Distance	.16652	- .01904
Density	.3130	- .72998*	Density	.38885*	- .69239
Log Distance	- .26956	- .76685*	Log Distance	- .6550*	- .50307
Log Density	- .93386*	- .11049	Log Density	- .89199*	.61483
San Diego			San Jose		
Distance	.78319*	- .06601	Distance	1.1192*	1.5163*
Density	.44357*	- .1098	Density	- .42345*	- .37456
Log Distance	- .30572	- .50045*	Log Distance	- .54370*	.03344
Log Density	- .64744*	.27054	Log Density	-1.9211*	3.9980*
Seattle			St. Louis		
Distance	1.25290*	2.07874*	Distance	.57101*	- .75097*
Density	3.4636*	12.93868*	Density	.95988*	.10341
Log Distance	-1.0723*	1.85521*	Log Distance	- .72327*	.50038
Log Density	.13713	.07343	Log Density	- .77608	.34022
Spokane			Syracuse		
Distance	.56267*	- .67887	Distance	1.17404*	.70181
Density	.30312	-1.06449*	Density	.82670*	.29783
Log Distance	- .46036*	- .62448	Log Distance	.083928	-1.09381*
Log Density	- .88573*		Log Density	-1.0598*	.01327
Tacoma			Toledo		
Distance	.82170*	- .003415	Distance	.6942*	- .27270
Density	.91253*	.17775	Density	5.90793*	35.143*
Log Distance	- .75836*	.86223*	Log Distance	- .51916*	- .41152
Log Density	- .42952*	- .24568	Log Density	- .64385*	1.2484*

Table 1 (Continued)

City	Skewness	Kurtosis	City	Skewness	Kurtosis
Tucson			Utica		
Distance	.52109*	- .30894	Distance	.90579*	.80819*
Density	.43238*	- .05855	Density	.96889*	- .28172
Log Distance	- .79630*	.21538	Log Distance	- .39833*	- .56056
Log Density	-1.21662*	.98203*	Log Density	.037929	- .011955
Washington, D.C.			Wichita		
Distance	.55445*	- .2736	Distance	.66615*	.05911
Density	1.3408*	.71177	Density	- .25898	- .54026
Log Distance	- .45407*	- .6585	Log Distance	-1.22969*	3.40604*
Log Density	.13638	- .72145*	Log Density	-2.0544*	4.8377*

* Significantly different from zero at the .05 level.

APPENDIX

Table 2

Summary Results for Specifications (26)-(31), Semi-log and Double-log for Fifty Cities in 1970

City	Semi-log β_1	β_2	β_3	R^2	Double-log β_1	β_2	β_3	R^2
Akron								
Specifications[a]								
#1	-2.43E-1	4.08E-3		.22	1.44E-1	-3.99E-1		.22
#2	-2.60E-5*	1.09E-15		.22	-2.83E-5	6.35E-4		.22
#3	9.39E-5	-3.86E-9	3.68E-14	.25	1.28E-4	1.65	-1.22E-1	.23
Baltimore								
#1	-4.06E-1*	7.22E-1*		.85	3.31E-1	-5.60E-1*		.83
#2	-3.37E-5*	1.83E-16*		.86	-1.06E-5*	-2.39E-3*		.82
#3	-7.21E-6	-2.24E-10*	6.86E-16*	.88	3.60E-5*	1.48*	-1.02E-1*	.85
Birmingham								
#1	-2.13E-1*	1.48E-3		.51	-1.44E-1	-3.07E-1		.51
#2	-1.92E-1*	8.45E-6		.50	-1.13E-1	-1.68E-1		.51
#3	-9.68E-2*	9.68E-2*	-3.65E-3*	.56	-1.13E-1	3.79E-2	-2.69	.51
Boston								
#1	-5.30E-1*	3.79E-2*		.30	-8.84E-1*	1.32E-1		.25
#2	-3.67E-5**	2.40**		.31	1.87E-5	-3.97E-3**		.26
#3	-7.42E-6	-6.20E-10	6.08E-15	.32	7.27E-5	7.62E-1	-5.90E-2	.28
Buffalo								
#1	-1.13E-1	-1.15E-2		.63	1.13*	-8.14E-1*		.66
#2	-1.86E-1*	-5.06E-4		.62	-4.01E-1*	2.37E-1*		.64
#3	4.31E-1	-1.24E-1*	6.50E-3*	.65	-4.01E-1*	7.91E-1	-2.91	.64
Chicago								
#1	8.55E-2	-7.31E-3		.08	1.32	-4.17E-1		.08
#2	3.02E-2	-2.88E-4		.09	-1.49E-1*	2.72E-1		.09
#3	3.65E-1	-4.34E-2	1.39E-3	.10	-1.49E-1*	2.24E-1	-1.46E-1	.09

[a]The β_1, β_2, β_3 are regression coefficients for specifications #1, #2, #3. Specifications #1, #2, #3 represent equations (26, 29), (27, 30), and (28, 31) for the semi- and double-log functions, respectively.

Table 2 (Continued)

City	Semi-log β_1	Semi-log β_2	Semi-log β_3	Semi-log R^2	Double-log β_1	Double-log β_2	Double-log β_3	Double-log R^2
Cincinnati								
#1	-3.79E-1*	2.11E-2*		.41	-6.51E-1*	-9.73E-4		.41
#2	-2.70E-1*	1.19E-3		.39	-1.46E-3	-2.15E-1*		.40
#3	-6.82E-1*	9.21E-2	-4.62E-3	.40	-1.46E-3	1.63E-1	1.11E-1	.40
Cleveland								
#1	1.91E-1*	-3.26E-2*		.40	1.14*	-6.51E-1*		.31
#2	3.68E-2	-1.94E-3*		.41	-3.84E-1*	3.37E-1*		.36
#3	-2.41E-2	1.23E-2	-2.65E-3	.41	-3.84E-1	1.99E-1	-9.87E-1	.36
Columbus								
#1	-1.67E-1*	3.02E-3		.18	-1.73E-1	-1.60E-1		.18
#2	-1.56E-5*	1.97E-16		.19	-7.89E-6	-7.73E-4		.18
#3	1.65E-5	-5.30E-10	2.74E-15	.19	-7.59E-5	-1.45	9.88E-2	.19
Dallas								
#1								
#2	-1.75E-6	-5.14E-18		.02	1.04E-6	-5.54E-4		.03
#3	-1.12E-5	1.24E-10	-4.62E-16	.02	-3.91E-5	-8.99E-1	6.15E-2	.08
Dayton								
#1	-6.87E-1*	4.81E-2*		.57	-1.47*	2.58E-1		.46
#2	-4.72E-5**	2.93E-15**		.59	2.90E-5*	-5.95E-3*		.49
#3	7.90E-6*	-1.14E-9**	9.50E-15	.62	2.50E-4*	3.77*	-2.72E-1*	.61
Denver								
#1	-3.32E-1*	1.12E-2		.42	-4.63E-1	-1.71E-1		.43
#2	-4.36E-5*	1.03E-15*		.63	-7.53E-6	-2.53E-3*		.55
#3	2.13E-5	-1.02E-9*	5.32E-15*	.68	7.16E-5*	1.40*	-1.02E-1*	.60
Detroit								
#1	-1.14E-1*	1.76E-3		.29	1.07*	-4.27E-1*		.31
#2	-8.66E-2*	8.62E-10*		.82	-3.92E-4*	-2.31E-1*		.80
#3	-1.66E-3*	3.01E-3	-3.00E-7	.83	-3.92E-4*	4.23E-1	-2.84E-1	.80
Flint								
#1	-3.66E-1*	-2.17		.53	1.95E-1	-6.77E-1*		.54
#2	-1.44E-5*	3.88E-16		.16	-2.40E-6	-6.43E-4		.12
#3	1.78E-5	-4.80E-10	2.38E-15	.19	6.69E-5**	1.40*	-9.80E-2*	.19

Table 2 (Continued)

City	Semi-log β_1	β_2	β_3	R^2	Double-log β_1	β_2	β_3	R^2
Fort Worth								
#1	-1.81E-1*	7.69E-3		.15	1.37E-1	-1.62E-1		.13
#2	-2.67E-5*	5.56E-16		.42	-6.59E-6	-1.89E-3		.43
#3	-6.64E-5	7.93E-10	-3.96E-15	.43	-1.42E-5	-1.19E-1	6.58E-3	.43
Houston								
#1	-1.40E-1*	-2.02E-4		.45	1.57*	-7.13E-1*		.45
#2	-1.54E-5**	1.53E-17		.45	-1.82E-5	8.62E-4		.45
#3	2.39E-5	-3.63E-10**	8.83E-16**	.51	-6.00E-6	4.23E-1	-2.71E-2	.46
Jacksonville								
#1	-5.45E-1*	2.12E-2*		.60	-2.54E-1	-3.60E-1		.56
#2	-7.70E-6	-7.88E-16		.52	-2.21E-5*	7.45E-4		.50
#3	-9.28E-5*	1.61E-9*	-9.56E-15*	.58	-1.63E-4*	-2.57*	1.81E-1*	.56
Kansas City								
#1	-1.22E-1*	1.45E-3*		.23	-5.99E-3	-1.77E-1		.23
#2	-1.13E-5**	5.88E-17*		.23	-8.41E-6	-2.80E-4		.23
#3	-1.45E-5	5.19E-11	-1.72E-16	.23	-7.70E-6	1.23E-2	-1.15E-3	.23
Los Angeles								
#1	-2.21E-2	1.89E-4		.02	-3.74E-1	7.42E-2		.03
#2	-9.99E-6	-1.24E-16		.33	-1.26E-5	-9.05E-5		.33
#3	-2.86E-5	2.35E-10	-9.78E-16	.33	-8.64E-5	1.51	1.03	.35
Louisville								
#1	-1.44E-1*	5.42E-4		.46	1.05E-1	-2.79E-1		.45
#2	-1.46E-1*	2.92E-5		.48	-1.19E-1*	-4.35E-2		.48
#3	-1.51E-1	7.41E-4	1.24E-6	.48	-1.99E-1*	-1.45	9.21	.48
Memphis								
#1	-3.67E-2	-1.13E-2		.52	2.41E-1	-3.55E-1*		.48
#2	2.42E-6	-2.62E-16		.08	-1.65E-5	2.51E-3		.09
#3	3.14E-5	-3.75E-10	1.19E-15	.09	-4.23E-5	-6.97E-1	4.93E-2	.09
Miami								
#1	-3.04E-1*	1.13E-2*		.52	-6.27E-1*	-5.53*		.53
#2	-2.26E-5	4.69E-16		.52	1.34E-6**	-2.51E-3**		.53
#3	-3.97E-5	2.57E-10	-6.36E-16	.53	6.67E-6**	1.03**	-9.71E-3**	.53

Table 2 (Continued)

City	Semi-log β_1	β_2	β_3	R^2	Double-log β_1	β_2	β_3	R^2
Milwaukee								
#1	-3.23E-1	9.66E-3		.49	-1.43	9.50E-2		.50
#2	-2.30E-1	1.92E-4		.51	-3.24E-2	-3.19E-1		.53
#3	-1.71	2.50	-1.26	.57	-3.24E-2	6.35	-4.55	.53
Nashville								
#1	-5.14E-1*	2.20E-2*		.67	-7.92E-1	-1.52E-1		.66
#2	-4.02E-5**	1.21E-15**		.67	-1.33E-6	-3.41E-3		.66
#3	-4.89E-5	1.70E-10**	2.79E-16**	.67	2.11E-5	3.89E-1	-3.08E-2	.67
New Haven								
#1	-6.51E-5*	1.69E-2		.74	-5.26E-1	-5.49E-1*		.74
#2	-5.80E-5**	1.12E-15		.74	-3.58E-5**	-1.56E-3		.74
#3	-9.03E-5**	8.72E-10	-5.36E-15	.74	-2.90E-5	8.08E-2	-7.51E-3	.74
New Orleans								
#1	-4.63E-1*	2.79E-2*		.38	-6.98E-1	-2.09E-2		.37
#2	-3.48E-1*	1.86E-3*		.39	1.07E-2	-2.62E-1*		.37
#3	-1.85E-1	-3.76E-2	4.29E-3	.39	1.07E-2	-9.97E-1	4.03E-1	.37
Oklahoma City								
#1	9.77E-2	-2.49E-2*		.33	-6.54E-2	-2.39E-1*		.25
#2	2.42E-6	-1.91E-15**		.36	-2.52E-5*	1.04E-3		.28
#3	-1.13E-4**	2.48E-9**	-1.68E-14**	.49	-1.01E-4**	-9.60E-1*	7.17E-2*	.33
Omaha								
#1	-2.55E-1*	1.36E-2		.15	-4.49E-1	-3.09E-3		.15
#2	-1.99E-5**	8.77E-16		.15	1.77E-6	1.62E-3*		.15
#3	-6.47E-6	-2.96E-10	2.64E-15	.15	-1.20E-5	-1.89	1.21E-2	.15
Philadelphia								
#1	-7.69E-2	-7.10E-3		.49	8.68E-1	-6.13E-1*		.46
#2	-1.29E-5*	-2.68E-16		.49	-2.85E-5*	1.62E-3		.48
#3	-1.80E-5	6.62E-11	-5.06E-16	.49	-5.89E-5	-7.01E-1	4.97E-2	.49
Phoenix								
#1	1.99E-1*	-2.52E-2*		.48	8.83E-1*	-5.17E-1*		.35
#2	6.31E-6	-1.30E-15		.49	-3.63E-5**	3.47E-3**		.42
#3	-1.45E-5	3.55E-10	-3.02E-15	.50	-1.12E-4**	-1.30**	9.57E-2**	.48

Table 2 (Continued)

City	Semi-log β_1	β_2	β_3	R^2	Double-log β_1	β_2	β_3	R^2
Pittsburgh								
#1	-2.51E-1	1.55E-2		.14	-8.63E-3	1.86E-1		.17
#2	-1.67E-5	7.87E-16		.13	1.17E-5	-2.62E-3		.16
#3	-1.49E-4*	3.33E-9*	-2.30E-14*	.21	-1.11E-4	-1.74	1.22E-1	.19
Portland								
#1	-2.07E-1*	5.81E-3			3.01E-2	-2.44E-1		.32
#2	-1.85E-5*	3.72E-16		.33	-1.16E-5	-2.91E-4		.32
#3	5.87E-6*	-4.37E-10*	2.56E-15*	.34	1.01E-5	3.47E-1	-2.50E-2	.32
Providence								
#1	-1.21E-2*	-9.91E-3		.42	2.37E-1	-3.44E-1*		.38
#2	-5.90E-6**	-5.19E-16		.43	-2.17E-5**	1.00E-3		.40
#3	-3.58E-5*	4.98E-10	-2.78E-15	.44	-6.06E-5*	-6.76E-1	4.90E-2	.42
Richmond								
#1	-1.54E-1	-3.68E-3		.52	3.62E-1	-5.07E-1*		.50
#2	-1.63E-5*	-2.05E-16		.53	-1.99E-5*	-3.23E-4		.52
#3	-1.00E-4	1.01E-9	-3.50E-15	.61	-7.85E-5*	-1.55	1.04E-1	.54
Rochester								
#1	-4.71E-1*	9.70E-3			1.61E-1	-6.49E-1*		.74
#2	-4.20E-5*	5.13E-16		.74	-2.62E-5*	-1.25E-3		.74
#3	-3.60E-5	-1.11E-10	1.07E-15	.74	3.36E-5	1.08	-7.70E-2	.74
Sacramento								
#1	-4.15E-2*	-4.31E-3		.41	1.23E-1	-2.62E-1*		.37
#2	-6.41E-6	-1.84E-16		.41	-1.47E-5**	5.12E-4		.39
#3	-2.56E-5	2.42E-10	-1.00E-15	.43	-3.47E-5	-3.95E-1	2.83E-2	.41
Salt Lake City								
#1	-2.11E-1*	7.55E-3		.26	-4.98E-1	-1.79E-3		.27
#2	-1.56E-4**	2.85E-16		.25	1.26E-6	1.80E-3		.27
#3	-4.96E-5*	5.82E-10	-2.44E-15	.27	-1.11E-5*	-2.04E-1*	1.27E-2**	.27
San Antonio								
#1	-2.64E-1*	7.00E-3		.46	1.89E-1	-3.61E-1*		.46
#2	-2.51E-5**	6.65E-16		.46	-1.80E-5	-1.67E-4		.46
#3	4.71E-5	-1.61E-9*	1.10E-14*	.49	3.93E-5	8.60E-1	-6.17E-2	.46

Table 2 (Continued)

City	Semi-log				Double-log			
	β_1	β_2	β_3	R^2	β_1	β_2	β_3	R^2
San Diego								
#1	-1.27E-1*	5.49E-3		.15	-3.18E-1	9.97E-3		.14
#2	-9.54E-6**	2.61E-16		.15	1.99E-6	-1.13E-3*		.14
#3	-1.61E-7	-1.43E-10	8.79E-16	.15	6.29E-6	9.10E-2	-7.42E-3	.14
San Jose								
#1	-8.11E-2	3.02E-3		.04	-2.64E-2	-5.94E-2		.04
#2	-7.59E-6	2.07E-16		.05	-2.33E-6	-2.39E-4		.04
#3	3.20E-5	-6.42E-10	3.02E-15*	.09	-2.53E-7	4.04E-2	-3.07E-3	.04
Seattle								
#1	-2.84E-1*	7.39E-3*		.50	-8.92E-1*	-1.11E-2		.55
#2	-2.11E-5**	1.97E-16**		.49	3.09E-6	-3.21E-6		.55
#3	-5.37E-5**	3.71E-10*	-8.97E-16*	.53	-3.58E-6	-1.44E-1	6.86E-3	.55
St. Louis								
#1	-6.51E-2	-5.65E-3		.59	7.59E-1	-5.37E-1*		.58
#2	-9.69E-6*	-2.49E-16		.60	-2.31E-5*	1.26E-3		.59
#3	-3.31E-5	3.07E-10	-1.38E-15	.61	-2.73E-5	-9.20E-2	7.63E-3	.59
Spokane								
#1	-3.92E-1*	1.20E-2		.38	-1.02E-1	-3.85E-1		.38
#2	-3.28E-5**	6.35E-16		.38	-1.53E-5	-1.34E-3		.37
#3	-3.84E-5	1.05E-10	8.24E-17	.38	4.57E-5	9.81E-1	-7.12E-2	.38
Syracuse								
#1	-5.79E-1*	8.81E-3		.85	-2.62E-1	-6.93E-1*		.85
#2	-5.38E-5*	4.91E-16		.85	-4.12E-5*	-8.66E-4		.85
#3	-4.56E-5*	-1.58E-10	1.29E-15	.85	-2.47E-5	2.29E-1	1.75E-2	.85
Tacoma								
#1	-6.59E-1*	5.67E-2*		.46	-6.44E-1*	2.04E-2		.41
#2	-4.39E-5*	4.10E-15*		.45	1.74E-5	-3.64E-3*		.42
#3	-8.06E-5*	9.79E-10	-3.14E-15	.47	9.24E-5*	7.35E-1*	-5.92E-2*	.47
Toledo								
#1	-2.06E-1	-1.41E-2		.62	2.23E-1	-6.77E-1*		.61
#2	-2.66E-5**	-8.58E-16		.63	-4.80E-5**	1.52E-3		.62
#3	-4.27E-5	3.43E-10	-2.74E-15	.63	-4.55E-5	3.41E-2	-9.65E-4	.62

Table 2 (Continued)

City	Semi-log β_1	β_2	β_3	R^2	Double-log β_1	β_2	β_3	R^2
Tucson								
#1	-3.96E-1*	2.44E-2		.20	-5.11E-1	-3.25E-2		.19
#2	-2.82E-5**	1.47E-15		.20	6.10E-6	-2.51E-3		.19
#3	-3.83E-5	2.15E-10	1.82E-16	.20	7.44E-5	9.71E-1	-7.26E-2	.21
Utica								
#1	-7.93E-1*	5.74E-2*		.49	-1.16*	6.03E-2		.50
#2	-5.84E-5*	4.30E-15*		.48	1.55E-5	-4.85E-3*		.50
#3	-1.26E-4*	1.99E-9	-1.15E-14	.49	-4.01E-6	-2.09E-1	1.07E-2	.50
Washington, D.C.								
#1	-4.26E-1*	1.92E-2*		.36	-2.34*	4.38E-1		.39
#2	-2.81E-5*	7.33E-16*		.35	2.06E-5	-5.91E-3*		.38
#3	-8.25E-5*	7.85E-10	-2.51E-16	.38	-3.77E-5	-1.40	8.93E-2	.39
Wichita								
#1	3.07E-1*	-7.65E-2*		.60	7.09E-2	-4.72E-1*		.45
#2	5.00E-6	-6.49E-15*		.60	-6.58E-5*	3.69E-3*		.54
#3	6.01E-6	-2.89E-11	-6.26E-15	.60	-1.93E-4*	-1.13*	9.00E-2*	.61

*, ** Significantly different from zero at the .10 and .05 level, respectively.

APPENDIX

Table 3

The Optimal λ Value with Confidence Region for λ for the Nine Alternative Specifications [equations 6-14] for Fifty Cities in 1970

City	Equation Number 6	7	8	9	10	11	12	13	14
Akron	-.10 .00~-.30[b]	-.50 -.20~-.50	-.10 .00~-.30	-.10 .00~-.30	-.10 .00~-.30	-.10 -.10~-.30	-.10 .10~-.30	-.10 .00~-.30	-.10 .00~-.30
Baltimore	.30 .40~.10	.60 1.20~.10	.20 .30~.10	.30 .40~.20	.20 .30~.20	.20 .30~.10	.00 .10~-.10	.20 .40~.10	.00 .10~-.10
Birmingham	.30 .50~.10	.50 1.00~.00	.50 .70~.30	.50 .70~.20	.40 .70~.20	.50 .70~.30	.40 .50~.20	.30 .50~.10	.40 .60~.20
Boston	.10 .40~-.20	.40 1.00~-.20	.20 .60~-.10	.10 .60~-.10	.30 .60~.00	.20 .60~-.10	.10 .40~-.10	.10 .50~-.10	.20 .50~-.10
Buffalo	.20 .50~.00	.70 1.50~-.40	.20 .50~.00	.30 .50~.00	.20 .50~.00	.30 .50~.10	.50 .70~.30	.30 .50~.10	.30 .50~.00
Chicago	-.10 .30~-.40	.30 .80~-.20	-.10 .30~-.40	-.10 .30~-.40	-.10 .30~-.40	-.10 .30~-.40	-.10 .30~-.40	-.10 .30~-.40	-.10 .30~-.40
Cincinnati	.20 .40~-.10	.10 .30~-.10	.30 .60~.10	.30 .60~.10	.30 .60~.10	.30 .60~.10	.20 .50~.10	.20 .50~.00	.30 .50~.10
Cleveland	.10 .40~-.30	.10 .60~-.50	.00 .40~-.30	.10 .50~-.20	.10 .40~-.30	.10 .50~-.20	.40 .70~.00	.20 .50~-.10	-.10 .30~-.50

[b]Confidence region for λ at the .05 level from equation 5.

Table 3 (Continued)

City	Equation Number 6	7	8	9	10	11	12	13	14
Columbus	.50 .90~.10	.40 .90~-.10	.50 .90~.10	.50 .90~.10	.50 .90~.10	.50 .90~.10	.50 .90~.10	.50 .90~.10	.50 .90~.10
Dallas	.50 .80~.30	.80 1.50~.30	.70 1.00~.40	.70 1.00~.40	.70 1.00~.40	.70 1.00~.40	.60 .80~.40	.60 .80~.40	.60 .90~.40
Dayton	.20 .60~-.10	.60 1.50~-.40	.40 .70~.10	.40 .70~.10	.40 .70~.10	.30 .60~.10	.20 .50~-.10	.30 .60~-.10	.30 .60~.10
Denver	.10 .30~.00	.20 .40~.10	.20 .40~.00	.20 .30~.00	.20 .40~.00	.20 .40~.10	.20 .40~.10	.20 .30~.00	.20 .40~.00
Detroit	.50 .70~.30	.30 1.50~-.50	.50 .70~.30	.50 .70~.20	.50 .60~.30	.40 .70~.20	.50 .70~.20	.50 .70~.30	.50 .70~.30
Flint	.00 .30~-.20	.10 .40~-.20	.10 .40~-.10	.10 .30~-.10	.10 .30~-.10	.10 .30~-.10	.20 .40~.00	.10 .30~-.10	.10 .30~-.10
Fort Worth	.60 1.00~.30	.10 .40~-.10	.60 1.00~.30	.60 1.00~.30	.70 1.00~.30	.60 1.00~.30	.60 1.00~.30	.60 1.00~.30	.60 1.00~.30
Houston	.50 .80~.30	.70 1.10~.40	.60 .90~.30	.60 .90~.40	.60 .90~.30	.60 .90~.40	.70 .90~.40	.60 .90~.40	.60 .90~.30
Jacksonville	.00 .20~-.20	.20 .30~.10	.10 .30~.00	.10 .30~.00	.10 .30~.00	.20 .30~.10	.30 .50~.20	.20 .30~.00	.10 .30~.00
Kansas City	.40 .70~.10	.90 1.50~.20	.40 .70~.10	.40 .80~.10	.40 .80~.10	.40 .80~.10	.50 .80~.10	.40 .80~.10	.40 .80~.10
Los Angeles	-.20 .30~-.50	-.10 .30~-.50	-.20 .30~-.50	-.10 .30~-.50	-.20 .20~-.50	-.20 .30~-.50	-.20 .20~-.50	-.20 .30~-.50	-.20 .30~-.50
Louisville	.10 .50~-.20	.00 .30~-.40	.20 .60~-.10	.20 .60~-.10	.20 .60~-.10	.30 .60~-.10	.30 .60~.10	.20 .50~-.10	.20 .60~-.10

Table 3 (Continued)

City	Equation Number 6	7	8	9	10	11	12	13	14
Memphis	.40 .60~.20	-.40 -.20~-.50	.50 .80~.30	.50 .80~.30	.50 .80~.30	.60 .90~.30	.50 .70~.30	.50 .70~.20	.50 .80~.20
Miami	.10 .40~-.10	-.50 .30~-.50	.30 .60~.10	.40 .60~.10	.30 .60~.10	.30 .60~.10	.20 .40~.10	.20 .40~.00	.30 .50~.10
Milwaukee	.00 .30~-.30	.20 .80~-.50	.30 .70 -.10	.30 .70~-.10	.30 .60~-.10	.30 .70~.10	.20 .50~-.10	.10 .40~-.20	.20 .60~-.20
Nashville	.20 .40~.00		.30 .50~.10	.30 .50~.10	.30 .50~.10	.30 .50~.10	.20 .40~.10	.20 .40~.00	.40 .50~.10
New Haven	.00 .20~-.20	.70 1.20~.20	.30 .60~.10	.30 .50~.10	.30 .60~.10	.30 .50~.10	.30 .40~.20	.10 .30~.00	.30 .50~.10
New Orleans	.10 .20~-.50	-.10 .20~-.40	.10 .20~-.50	.10 .20~-.50	.10 .20~-.50	.10 .20~-.50	.10 .20~-.50	.10 .20~-.50	-.20 .10~-.40
Oklahoma City	.70 1.10~.40	.80 1.10~.40	.80 1.10~.40	.80 1.10~.40	.80 1.10~.40	.80 1.10~.40	.80 1.10~.40	.80 1.10~.40	.70 1.10~.40
Omaha	.10 .20~-.10	.30 .60~.00	.10 .20~-.10	.10 .20~-.10	.10 .20~-.10	.10 .20~-.10	.10 .20~-.10	.10 .20~-.10	.10 .20~-.10
Philadelphia	.10 .30~-.20	.40 .90~-.10	.20 .40~-.10	.20 .40~-.10	.20 .40~-.10	.20 .50~.10	.30 .50~.10	.20 .40~.00	.30 .50~-.20
Phoenix	.80 1.30~.50	.20 .30~-.50	.80 1.30~.50	.90 1.30~.50	.80 1.30~.50	.90 1.40~.50	1.00 1.40~.70	.90 1.30~.50	.80 1.20~.40
Pittsburgh	.40 .90~.00	-.30 .00~-.50	.50 1.10~.10	.60 1.10~.10	.50 .90~.00	.60 1.10~.10	.40 .90~.00	.40 .90~.00	.50 1.00~.00
Portland	.50 .80~.10	-.10 .20 -.50	.50 .90~.20	.50 .90~.20	.50 .90~.20	.50 .90~.20	.60 .90~.30	.50 .90~.10	.50 .90~.20

Table 3 (Continued)

City	Equation Number 6	7	8	9	10	11	12	13	14
Providence	.80 1.30~.50	.40 1.00~-.30	.80 1.30~.50	.90 1.30~.50	.80 1.30~.50	.90 1.40~.50	1.00 1.40~.70	.90 1.30~.50	.80 1.20~.40
Richmond	.00 .20~-.10	.30 .90~-.40	.20 .30~.00	.20 .30~.00	.20 .30~.00	.20 .30~.10	.20 .30~.10	.10 .30~.00	.10 .30~.00
Rochester	.20 .30~.00	-.20 .20~-.50	.30 .50~.10	.30 .40~.10	.30 .50~.20	.30 .40~.10	.30 .40~.20	.20 .40~.10	.30 .50~.20
Sacramento	.50 .80~.20	.80 1.50~.30	.50 .90~.30	.60 .90~.30	.50 .90~.30	.60 .90~.30	.60 .90~.40	.50 .80~.30	.50 .90~.30
Salt Lake City	.60 .90~.30	-.20 .20~-.50	.70 1.00 .40	.70 1.00~.40	.70 1.00~.40	.70 1.00~.40	.60 .90~.30	.60 1.00~.30	.70 1.00~.40
San Antonio	.50 .90~.30	.20 .40~.00	.50 .90~.30	.50 .90~.30	.50 .90~.30	.50 .90~.30	.60 .90~.30	.50 .90~.30	.50 .90~.30
San Diego	.40 1.00~-.10	.00 .30~-.30	.50 1.10~-.10	.50 1.10~.00	.50 1.10~-.10	.50 1.10~.00	.50 1.00~.00	.50 1.00~-.10	.50 1.10~-.10
San Jose	1.10 1.50~.70	1.20 1.50~.60	1.10 1.50~.70	1.10 1.50~.70	1.10 1.50~.70	1.10 1.50~.70	1.20 1.50~.70	1.10 1.50~.70	1.10 1.50~.60
Seattle	.10 .20~-.30	-.10 .10~-.30	.20 .40~.00	.20 .40~.00	.20 .30~.00	.20 .40~.10	.10 .20~.00	.00 .20~-.20	.10 .20~-.10
St. Louis	.10 .30~-.10	-.50 1.20~-.50	.10 .40~.00	.20 .40~.00	.20 .40~.00	.20 .40~.10	.30 .50~.20	.20 .40~.00	.20 .40~.00
Spokane	.50 .80~.30	-.20 .50~-.50	.60 .80~.30	.60 .80~.30	.60 .80~.30	.60 .80~.30	.60 .80~.40	.60 .80~.30	.60 .80~.30
Syracuse	.10 .30~.00	.00 .40~-.40	.30 .50~.20	.30 .50~.20	.40 .50~.20	.30 .50~.20	.30 .40~.30	.20 .30~.10	.30 .40~.20

Table 3 (Continued)

City	Equation Number 6	7	8	9	10	11	12	13	14
Tacoma	.40 .70~.00	-.10 .10~-.30	.60 .90~.20	.50 .90~.20	.60 .90~.30	.50 .80~.20	.30 .60~.10	.40 .80~.10	.50 .80~.20
Toledo	.00 .10~-.10	-.20 .00~-.40	.00 .10~-.10	.00 .10~-.10	.00 .10~-.10	.10 .10~-.10	.10 .20~.00	.00 .10~-.10	.00 .10~-.10
Tucson	.70 1.00~.40	.10 .40~-.20	.70 1.00~.40	.70 1.00~.40	.70 1.00~.40	.70 1.00~.40	.70 1.00~.40	.70 1.00~.40	.70 1.00~.40
Utica	.10 .30~-.40	.10 .30~-.30	.10 .40~-.20	.10 .40~-.20	.10 .40~-.20	.10 .40~-.20	.10 .30~-.20	.00 .30~-.30	.00 .30~-.20
Washington, DC	.00 .30~-.20	.00 .50~-.50	.10 .40~-.10	.10 .40~-.10	.10 .40~-.10	.10 .40~-.10	.10 .40~-.10	.10 .30~-.20	.10 .40~-.10
Wichita	.80 1.30~.50	.10 .40~-.30	.80 1.30~.50	.80 1.30~.50	.80 1.30~.50	.90 1.50~.50	1.00 1.50~.60	.80 1.40~.40	.80 1.30~.30

APPENDIX

Table 4

Generalized Functional Forms and the Corresponding Integral for Predicting Population with Comparisons of the Simple Functional Forms[a]

Equation 6

Simple Exponential $\ln D(u) = \beta_1 + \beta_2 u$

Generalized Exponential $D(u) = [\beta_1 + \beta_2 u]^{1/\lambda}$

Simple Integral $N = \emptyset \exp(\beta_1)\beta_2^{-2}[1 - (1 - \beta_2\bar{u})\exp(\beta_2\bar{u})]$

Generalized Integral

$$N = \frac{\emptyset(\beta_2)^{-2}}{(1/\lambda+1)(1/\lambda+2)}\left\{[\beta_1 + \beta_2\bar{u}]^{1/\lambda+1}(\beta_2\bar{u}(1/\lambda+1) - \beta_1) + \beta_1^{1/\lambda+2}\right\}$$

Equation 7

Simple Binomial Approx. $\ln D(u) = \beta_1 + \beta_2 \ln(\bar{u}-u)$

Generalized Binomial Approx. $D(u) = [\beta_1 + \beta_2 (\bar{u}-u)^{\lambda}]^{1/\lambda}$

Simple Integral

$$N = \frac{\emptyset e^{\beta_1}}{(\beta_2+1)(\beta_2+2)}\left[\bar{u}^{\beta_2+2}\right]$$

Generalized Integral

$$N = \emptyset \int e^{y}(e^{y} - \bar{u})[\beta_1 + \beta_2 (y)^{\lambda}]^{1/\lambda} dy$$

where $y = \ln(\bar{u} - u)$

This integral can be simplified if $1/\lambda$ is an integer, say $1/\lambda = n$.

This is done by the expansion

$$[\beta_1 + \beta_2 y^{1/n}]^n = \sum_{i=0}^{n} \binom{n}{i} \beta_1^{n-i} \beta_2^{i} y^{i/n}$$

and then perform the integration to obtain cumulative γ-functions.

Need $\beta_1 + [\beta_2(\ln(\bar{u} - u))^{\lambda}]^{1/\lambda} > 0$ always unless $1/\lambda$ is a positive even integer.

Also $(\bar{u} - u) \geq 0$ and $(\bar{u} - u) \geq 0$ if λ is not a positive even integer.

Table 4 (Continued)

Equation 8

Simple Standardized Normal $\ln D(u) = \beta_1 + \beta_2 u^2$

Generalized Standardized Normal $D(u) = [\beta_1 + \beta_2 u^2]^{1/\lambda}$

Simple Integral $$N = \frac{\phi e^{\beta_1}}{2\beta_2} e^{\beta_2 (\bar{u})^2} - 1$$

Generalized Integral $$N = \frac{\phi}{2\beta_2 (1/\lambda+1)} \{(\beta_1 + \beta_2 \bar{u}^2)^{1/\lambda+1} - \beta_1^{1/\lambda+1}\}$$

Equations 9* and 11

Simple Gamma $\ln D(u) = \beta_1 + \beta_2 \ln(u) + \beta_3 u$

Generalized Gamma $D(u) = [\beta_1 + \beta_2 u^{\lambda} + \beta_3 u]^{1/\lambda}$

Simple Integral $$N = \phi e^{\beta_1} |\beta_3|^{-(\beta_2+2)} \Gamma(\beta_2 + 2) F\gamma^*(|\beta_3|\bar{u}|\beta_2 + 2)$$

Generalized Integral: If $\lambda = 1/n$ for n an integer, then

$$N = \phi \sum_{i=0}^{n} \sum_{j=0}^{i} \binom{n}{i} \binom{i}{j} \beta_2^{n-i} \beta_1^{i-j} \beta_3^{j} \left. \frac{u^{2+j+(n-i)/n}}{2+j+(n-i)/n} \right|_{\underline{u}}^{\bar{u}}$$

Equation 10

Simple Alternative Function $\ln D(u) = \beta_1 + \beta_2 u + \beta_3 (1/u)$

Generalized Alternative Function $D(u) = [\beta_1 + \beta_2 u + \beta_3 (1/u)]^{1/\lambda}$

Simple Integral: This class of integrals has no solutions in closed form.

Generalized Integral

If $\lambda = 1/n$ for n an integer, then

$$N = \phi \sum_{i=0}^{n} \sum_{j=0}^{i} \binom{n}{i} \binom{i}{j} \beta_2^{n-i} \beta_1^{j} \beta_3^{i-j} \left. \frac{u^{2+j+n-2i}}{2+j+n-2i} \right|_{\underline{u}}^{\bar{u}}$$

This may not converge for $\underline{u}=0$ and when $2+j+n-2i=0$ one must replace $\frac{u^{2+j+n-2i}}{2+j+n-2i}$ with $\ln u$.

Table 4 (Continued)

Equation 12

No solution in generalized form; must use numerical analysis.

Equation 13

Simple Gamma with b=0 $\quad \ln D(u) = \beta_1 + \beta_2 \ln(u)$

Generalized Gamma with b=0 $\quad D(u) = [\beta_1 + \beta_2 u^{\lambda}]^{1/\lambda}$

Simple Integral $\quad N = \frac{\phi e^{\beta_1}}{\beta_2+2} \bar{u}^{\beta_2+2}$

Generalized Integral: This integral can be solved if $1/\lambda$ is an integer, $1/\lambda = n$

$$N = \phi \sum_{i=0}^{n} \binom{n}{i} \frac{\beta_1^{n-i}}{2+i/n} \beta_2^{i} \bar{u}^{2+i/n} \quad \text{where } \binom{n}{i} = \frac{n!}{(n-i)!\,i!}$$

Equation 14*

Simple General Normal $\quad \ln D(u) = \beta_1 + \beta_2 u + \beta_3 u^2$

Generalized General Normal $\quad D(u) = [\beta_1 + \beta_2 u + \beta_3 u^2]^{1/\lambda}$

Simple Integral $\quad N = \frac{\phi e^{(\beta_1 - \beta_2 t^2)}}{2\beta_3} \{ e^{\beta_3 (\bar{u}+t)^2} - e^{\beta_3 t^2} - \beta_2 \int_t^{\bar{u}+t} e^{\beta_3 \nu^2} d\nu \}$

where $\nu = u-t$ and $t = \beta_2/2\beta_3$

Generalized Integral

$$N = \frac{\phi}{2\beta_3 \lambda} (\lambda+1) \{ (\beta_1 + \beta_2 \bar{u} + \beta_3 \bar{u}^2)^{1/\lambda+1} - \beta_1^{1/\lambda+1} \}$$

[a]The equation numbers correspond to the main text.

*The equations marked with a star correspond to the equations in the McDonald-Bowman (12) paper which seem to be in error.

BOUND BY
HM

UNIVERSITY OF ILLINOIS-URBANA
3 0112 060296594